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HAE-003-001617 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

June / July - 2017

Mathematics : Paper - 602 (A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003
Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following : **20**

(1) Define Compact set.

(2) Define Connected set.

(3) State Heine-Borel Theorem.

(4) Is Z (Set of Integer) compact set ?

(5) Is the arbitrary intersection of compact set compact ?

(6) L (sinbt) = ____________.

(7) Define $L\left(e^{at}t^{n}\right) =$ ____________.

(8) Define convolution function.

(9) $L\left(f\left(t\right)\right) = \overline{f}\left(s\right)$ then $L\left[\int_0^t f\left(u\right)du\right] =$ ____________.

(10) $L^{-1}\left(\frac{1}{4s+5}\right) =$ ____________.

(11) Define Nilpotent element.

(12) How many proper Ideal can a field have ?

(13) Define Integral Domain.

(14) Define Divisor of Polynomial.

(15) Find characteristic of the ring $\left(Z_{6,}\ +_{6,}\ *_{6}\right)$.

(16) Find the greatest lower bound of $\left\{\frac{1}{n} \middle| n \in N\right\}$.

(17) Find $L^{-}\left(\frac{1}{s^3}\right) =$ ____________.

(18) If polynomial $f = (3, 0, 0, 0.........)$ then find [f]

(19) Find zero divisor of $(Z_{6}, +_{6}, *_{6})$.

(20) What do you mean by a Cubic polynomials ?

2 (a) Attempt any three : **6**

(1) Determine the subset $(-\infty, 0) \cup \{3\}$ of metric space R is Open, Closed, Connected or Compact.

(2) Show that A = (2, 4] and B = (4, 5) are note Separated sets of Metric space R.

(3) State and prove Heine-Borel Theorem.

(4) Prove if $L\{f(t)\} = \overline{f(s)}$ then $L\{e^{at} f(t)\} = \overline{f(s-a)}$.

(5) Prove if $L\{f(t)\} = \overline{f(s)}$ then $L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\overline{f(s)}$.

(6) Find Inverse Laplace Transformation of $\frac{3s+5}{(s+1)^4}$.

(b) Attempt any three : **9**

(1) Prove that Every Closed subset of Compact set is Compact in metric space.

(2) Show that the set of all odd natural number is countable set.

(3) Let (X, d) be a metric space. Then X is totally bounded set.

(4) Find Laplace transformation of $\frac{\cos 2t - \cos 3t}{t}$.

(5) Let $L\{f(t)\} = \overline{f(s)}$ then $L\left(t^n f(t)\right) = (-1)^n \frac{d^n}{ds^n}\left[\overline{f(s)}\right]$ where n = 1, 2, 3.......

(6) Find $L^{-1}\left\{\frac{1}{s(s+a)^3}\right\}$.

(c) Attempt any two : **10**

(1) Prove that every compact subset of metric space is closed in metric space.

(2) State and prove theorem of Nested intervals.

(3) For the set $E = \left[\frac{1}{3}, \frac{2}{3}\right]$ the collection $\{G_n \mid n \in N\}$ where $G_n = \left(\frac{1}{n}, 1\right)$ is a cover of E or not. Is it an open cover of R.

(4) Using Convolution theorem, find inverse Laplace Transformation of $\frac{s^2 - a^2}{\left(s^2 + a^2\right)^2}$.

(5) Find Laplace transformation of $te^{-3t} \sin^2 t$.

3 (a) Attempt any three : **6**

(1) $\Phi : (G, *) \to (G', \Delta)$ be Homomorphism. If N' is a Normal subgroup of $\Phi(G)$ then $\Phi^{-1}(N)$ is a Normal subgroup of G.

(2) Show that a cyclic group of order eight is homomorphism to a cyclic group of order four.

(3) $I = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \middle| a, b \in Z \right\}$ is an Ideal ?

(4) Prove that field has no proper Ideal.

(5) $f(x) = (2, 3, 4, 2, 0, 0, ...)$ and $g(x) = (4, 2, 0, 0, 3, 0..........) \in Z_5[x]$ then find $f(x) \cdot g(x)$.

(6) Factorize $f(x) = x^4 + 4 \in Z_5[x]$ by using factor theorem.

(b) Attempt any three : **9**

(1) For $n \geq 2$ the alternating group A_n of S_n is a normal subgroup of S_n.

(2) Let $\Phi : (G, *) \to (G', \Delta)$ be Homomorphism. Then K_ϕ is a normal subgroup of G.

(3) Is $R = \left\{ \frac{m}{2^n} \,\middle|\, m, n \in Z \right\}$ a ring with respect to usual addition and multiplication ?

(4) Prove that field has no proper ideal.

(5) Find g.c.d. of $f(x) = x^3 + 3x^2 + 3x + 3$ and $g(x) = 4x^3 + 2x^2 + 2x + 2 \in Z_5[x]$.

(6) Let $f(x), g(x), p(x) \in F[x]$. If $p(x)/f(x)$ and $p(x)/g(x)$ then, $p(x)/\{a(x)f(x) + b(x)g(x)\}$, $\forall = a(x), b(x) \in F[x]$

(c) Attempt any three : **10**

(1) State and prove First Fundamental theorem of Homomorphism.

(2) A commutative ring with unity is a field if it has no Proper Ideal.

(3) A non-empty subset I of a ring R is an Ideal of R. iff the following two conditions hold.

(a) $a - b \in I \quad \forall a, b \in I$

(b) $ra, ar \in I, \quad \forall a \in R$

(4) Express $f(x)$ as $q(x)g(x) + r(x)$ form by using division algorithm for

$$\left.\begin{array}{l} f(x) = x^6 - 3x^5 + 4x^2 - 3x + 2 \\ g(x) = x^2 + 2x - 3 \end{array}\right] \in Z_5[x]$$

(5) Any Ideal in integral domain $F[x]$ is a Principal Ideal.
